Crossian Resonance

A Multidisciplinary Research Journal **ISSN 0976-5417**
(A refereed Biannual Published in June and December)
Vol. 15, No.2, December 2024
https://research.holycrossngl.edu.in/

Relatively Prime Domination in Power of Coconut Tree Graph

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ABSTRACT

A subset S of V is said to be dominating set in G if every vertex in $V - S$ is adjacent to at least one vertex in S. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that $(d(u), d(v)) = 1$ where d is the degree of the vertex. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. If there is no such pair exist, then $\gamma_{rpd}(G) = 0$. This article focuses on exploring the relatively prime domination number within the context of the power of the coconut tree graph, denoted as $CT(m,n)$. The discussion reveals that for the coconut tree graph $CT(m,n)$, the relatively prime domination number, denoted as $\gamma_{rpd}(CT(m,n))$, assumes values of 0, 2, or 3. Additionally, the article describes the computation of the relatively prime domination number for the power of coconut tree graph using the Python programming language.

Keywords: *Dominating Set, Domination Number, Relatively Prime Dominating Set, Relatively Prime Domination Number*

1 Introduction

By a graph G = (V, E) we mean a finite undirected graph without loops and multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms, we refer to Harary [1] and for terms related to domination we refer to Haynes [2]. A subset S of V is said to be a dominating set in G if every vertex in V– S is adjacent to at least one vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set in G. Berge [3] and Ore [4] formulated the concept of domination in graphs. It was further extended to define many other dominations related parameters in graphs. In 2017, C. Jayasekaran and A. Jancy Vini [5] have introduced the concept of relatively prime domination number in graph theory. Let G be a non – trivial graph. A set $S \subseteq V$ is said to be a relatively prime dominating set if it is a dominating set and for every pair of vertices u and v in S such that (d(u), d(v)) = 1. The minimum cardinality of a relatively prime dominating set is called the relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. A Coconut

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Tree CT (m, n) is the graph obtained from the path P_n by appending m new pendent edges at an end vertex of P_n. The k^{th} power G^k of an undirected graph G is a graph that has the same set of vertices, but in which two vertices are adjacent when their distance in G is at most k. In this paper we determine the relatively prime domination number for power of coconut tree graph when $m = 2, 3$. In order to find the relatively prime domination number, we must know the degree of each vertex in the graph. The following results exhibit the degree of vertices in the coconut tree graph and power of coconut tree graph.

2 Distribution of Degree of Vertices

2.1 Coconut Tree Graph

We shall now discuss the distribution of degree of vertices in the Coconut Tree graph. Here, we give name to the end vertex of pendent edges as $v_1, v_2, \ldots., v_m$ and the vertices of the path as $u_1, u_2, \ldots., u_n$. All the end vertices $v_1, v_2, \ldots., v_m, u_n$ have degree one, the vertex u_1 which is adjacent to all m vertices and u_2, so that $u_1$have degree $(m + 1)$ and all other vertices $u_2, u_3, \ldots, u_{n-1}$ have degree two.

For example, consider the coconut tree graph given below

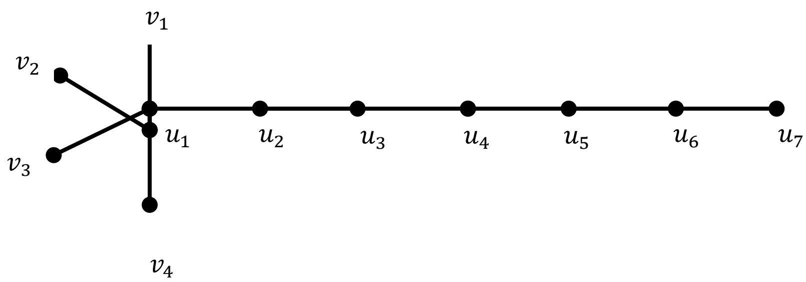


Figure 1

Here, v_1, v_2, v_3, v_4, u_7 have degree one, u_1 have degree five, and all other vertices $u_2, u_3, \ldots, u_6$ have degree two.

2.2 Power of Coconut Tree Graph

The power of the Coconut Tree graph follows a different distribution of degrees. Now, let us assume the power of the Coconut Tree be p. The vertices $v_1, v_2, \ldots., v_m$ have degree $m + p - 1$ since each vertex v_m is adjacent to all other v_{m-1} vertices and also adjacent to p vertices of P_n. When $n \geq 2p,$ the vertex u_1 is adjacent to each v_m vertices and to p vertices of P_n so that the degree of u_1 is $m + p$. Similar to u_1 the vertex u_2 is adjacent to $m + p$ vertices and also to u_1. Therefore, the degree of u_2 is $m + p + 1$. Proceeding like this till u_p we have $deg(u_i) = m + p + (i - 1)$ where $1 \leq i \leq p$.

When $n = p + 1$, the vertices $u_1, u_2, \ldots, u_p$ have the same degree $m + p$. Each vertex u_i where i=1,2,….p is adjacent to every other vertices of the path since the distance between them is at most p. The remaining vertex u_n have degree p since it is adjacent to p vertices to the left.

When $n = p + i$ where $2 \leq i \leq p - 1$, the vertices $u_i, u_{i+1}, \dots, u_p$ have same degree $m + p + (i - 1)$ for the graph with $n = p + i$ since the number of vertices to the right of each vertex is decreasing by 1 when i increasing. The vertex u_1 is adjacent to each v_m vertices and to p vertices of P_n so that the degree of u_1 is $m + p$. Similar to u_1, the vertex u_2 is adjacent to $m + p$ vertices and also to u_1. Therefore, the degree of u_2 is $m + p + 1$. Proceeding like this till u_{i-1} we have $deg(u_j) = m + p + (j - 1)$ where $1 \leq j \leq (i - 1)$.

These conditions follow until u_p because the vertices $v_1, v_2, \dots, v_m$ are adjacent to u_p^{th} vertex of P_n. And the remaining vertices of the graph with $n \geq 2p$, $u_{p+1}, u_{p+2}, \dots, u_{n-p}$ have degree $2p$ since each vertex covers p vertices on each side of it in the path P_n. For the graph with $n = p + 1$ and $n = p + i$, where $2 \leq i \leq p - 1$ have no vertices of degree $2p$, since $n < 2p$ and the remaining vertices u_k where $n - p + 1 \leq k \leq n$ has $deg(u_k) = 2p - l$ where $1 \leq l \leq p$.

Theorem 2.1. For any CT (m, n) graph $\gamma_{rpd}(CT(m,n)^p) = 0$ if $2 \leq n \leq p$ where $p \geq 2$.

Proof: Let G be a $CT(m,n)^p$ graph. As the power of G is a complete graph, the degree of each vertex is the same.

i.e.) Degree of each vertex is m+n-1. Hence a relatively prime dominating set does not exist.

$\therefore \gamma_{rpd}\left(\left(CT(m,n)\right)^p\right) = 0$ if $2 \leq n \leq p$.

3 Relatively Prime Domination Number on Power of Coconut Tree Graph when $m = 2$

This section covers the relatively prime domination number on power of coconut tree graphs when the number of pendant edges is 2. The relatively prime domination number for such graphs will be 2 or 3 or 0.

Theorem 3.1. For any Coconut Tree $CT(2,n)$ graph,

$\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 2$ if $p + 1 \leq n \leq 4p + 1$ where $p \geq 2$.

Proof: Let G be a Coconut Tree $\left(CT(2,n)\right)^p$ graph. We shall prove this by two cases.

Case 1: $p + 1 \leq n \leq 2p$ where $p \geq 2$

As the graph G has $p + i + 2$ vertices and u_p has degree $p + i + 1$ where $1 \leq i \leq p$, the vertex u_p covers all the vertices of G. Since the relatively prime dominating set contains at least two vertices. We proceed by two cases,

Subcase 1.1: $n = p + 1$

We choose two vertices u_p and v_i where $i = 1\ or\ 2$.

$\therefore \{u_p, v_i\}$ is a dominating set of G.

Since $\left(d(u_p), d(v_i)\right) = (p+2, p+1) = 1$, the relatively prime dominating set is $\{u_p, v_i\}$ and hence $\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 2$.

Subcase 1.2: $p + 2 \leq n \leq 2p$

We choose two vertices u_p and u_{p-i} where i=p-1, p-2, …..,2,1 for the graph with $n = p+2, p+3, \ldots .2p$ respectively.

$\therefore \{u_p, u_{p-i}\}$ is a dominating set of G.

Since $\left(d(u_p), d(u_{p-i})\right) = (p+1+j, p+j) = 1$ where $2 \leq j \leq$p, the relatively prime dominating set is $\{u_p, u_{p-i}\}$ and hence $\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 2$

Case 2: $2p + 1 \leq n \leq 4p + 1$

As the graph $(CT(2,n))^p$ has $n+2$ vertices and the vertex u_p has degree $2p+1$, the vertex u_p covers $v_1, v_2, u_1, u_2, \ldots, u_{p-1}, u_{p+1}, \ldots, u_{2p}$. Since $n = 2p + i$ where $1 \leq i \leq 2p + 1$, the graph has i vertices remaining and the vertex u_{n-p} has degree $2p$. The vertex u_{n-p} covers all the remaining i vertices.

$\therefore \{u_p, u_{n-p}\}$ is a dominating set of G.

Since $\left(d(u_p), d(u_{n-p})\right) = ((2p+1), (2p)) = 1$, the relatively prime dominating set is $\{u_p, u_{n-p}\}$ and hence $\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 2$.

Theorem 3.2. For any $CT(2,n)$ graph, $\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 3$ if $4p + 2 \leq n \leq 6p + 1$ where $p \geq 2$.

Proof: As the graph $(CT(2,n))^p$ has $n+2$ vertices and the vertex u_p has degree $2p+1$, the vertex u_P covers $v_1, v_2, u_1, u_2, \ldots, u_{p-1}, u_{p+1}, \ldots, u_{2p}$. Since $n = 2p + i$ where $2p + 2 \leq i \leq 4p + 1$, the graph has i vertices remaining and the vertex u_{3p+1} has degree $2p$. The vertex u_{3p+1} covers $u_{2p+1}, \ldots, u_{3p}, u_{3p+2}, \ldots., u_{4p+1}$. Since $n = 4p + 1 + j$, where $1 \leq j \leq 2p$, **t**he graph has remaining j vertices and the vertex u_{n-p+1} has degree $2p - 1$. The vertex u_{n-p+1} covers the remaining vertices j.

$\therefore \{u_p, u_{3p+1}, u_{n-p+1}\}$ is a dominating set.

Since $\left(d(u_p), d(u_{n-p+1})\right) = ((2p+1), (2p-1)) = 1$, $\left(d(u_p), d(u_{3p+1})\right) = ((2p+1), (2p)) = 1$, $\left(d(u_{3p+1}), d(u_{n-p+1})\right) = ((2p), (2p-1)) = 1$, $\left(d(u_p), d(u_{3p+1}), d(u_{n-p+1})\right) = 1$, the relatively prime dominating set is $\{u_p, u_{3p+1}, u_{n-p+1}\}$.

$\therefore \gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 3$.

Theorem 3.3. For any $CT(2,n)$ graph, $\gamma_{rpd}\left(\left(CT(2,n)\right)^p\right) = 0$ if $n \geq 6p + 2$ where $p \geq 2$.

Proof: Let G be a $(CT(2,n))^p$ graph. By theorem 3.2, $6p+1$ vertices are covered using u_p, u_{3p+1}, u_{n-p+1} vertices. But to cover the remaining vertices, we have to choose a vertex u_i of degree $2p$ where $3p+2 \leq i \leq n-p$. Then, $\left(d(u_{3p+1}), d(u_i)\right) = ((2p),(2p))) \neq 1$. Relatively prime dominating set does not exist.

$\therefore \gamma_{rpd}\left((CT(2,n))^p\right) = 0$ if $n \geq 6p+2$.

4 Relatively Prime Domination Number on Power of Coconut Tree Graph when $m = 3$

This section covers the relatively prime domination number on power of coconut tree graphs when the number of pendant edges is 3. The relatively prime domination number for such graphs will be 2 or 3 or 0.

Theorem 4.1. For any Coconut Tree $CT(3,n)$ graph, $\gamma_{rpd}\left((CT(3,n))^p\right) = 2$ if $p+1 \leq n \leq 4p$ where $p \geq 2$.

Proof: Let G be the $(CT(3,n))^p$ graph. We shall prove this by two cases.

Case 1: $p+1 \leq n \leq 2p$ where $p \geq 2$

As the graph G has $p+i+3$ vertices and u_p has degree $p+i+2$ where $1 \leq i \leq p$, the vertex u_P covers all the vertices of G. Since the relatively prime dominating set contains at least two vertices. We proceed by two cases,

Subcase 1.1: $n = p+1$

We choose two vertices u_p and v_i where $i = 1,2,3$.

$\therefore \{u_p, v_i\}$ is a dominating set of G.

Since $\left(d(u_p), d(v_i)\right) = (p+3, p+2) = 1$, the relatively prime dominating set is $\{u_p, v_i\}$ and hence $\gamma_{rpd}\left((CT(3,n))^p\right) = 2$.

Subcase 1.2: $p+2 \leq n \leq 2p$

We choose two vertices u_p and u_{p-i} where i=p-1, p-2, …..,2,1 for the graph with $n = p+2, p+3, \ldots .2p$ respectively.

$\therefore \{u_p, u_{p-i}\}$ is a dominating set of G.

Since $\left(d(u_p), d(u_{p-i})\right) = (p+j+2, p+j+1) = 1$ where $2 \leq j \leq$p, the relatively prime dominating set is $\{u_p, u_{p-i}\}$ and hence $\gamma_{rpd}\left((CT(3,n))^p\right) = 2$

Case 2: $2p+1 \leq n \leq 4p$ where $p \geq 2$

As the graph G has $2p+i+3$ vertices where $1 \leq i \leq 2p$ and the vertex u_{p-1} has degree $2p+1$, the vertex u_{p-1} covers $v_1, v_2, v_3, u_1, u_2, \ldots, u_{p-2}, u_p, \ldots., u_{2p-1}$. Since $n =$

$2p + j - 1$ where $2 \le j \le 2p + 1$, the graph has j vertices remaining and the vertex u_{n-p} has degree $2p$. The vertex u_{n-p} covers all the remaining j vertices.

$\therefore \{u_{p-1}, u_{n-p}\}$ is the dominating set of G.

Since $\left(d(u_{p-1}), d(u_{n-p})\right) = ((2p+1),(2p)) = 1$, the relatively prime dominating set is $\{u_{p-1}, u_{n-p}\}$.

$\therefore \gamma_{rpd}\left((CT(3,n))^p\right) = 2$ if $2p + 1 \le n \le 4p$ where $p \ge 2$

Theorem 4.2. For any $CT(3,n)$ graph, $\gamma_{rpd}\left((CT(3,n))^p\right) = 3$ if $4p + 1 \le n \le 6p$ where $p \ge 2$.

Proof: Let G be a $(CT(3,n))^p$ graph. As the graph G has $4p + i + 3$ vertices where $1 \le i \le 2p$ and the vertex u_{p-1} has degree $2p + 1$, the vertex u_{p-1} covers $v_1, v_2, v_3, u_1, u_2, \dots, u_{p-2}, u_p, \dots, u_{2p-1}$. Since $n = 2p + j - 1$ where $2 \le j \le 2p + 1$, the graph has j vertices remaining and the vertex u_{3p} has degree $2p$. The vertex u_{3p} covers $u_{2p}, u_{2p+1}, \dots, u_{3p-1}, u_{3p+1}, \dots, u_{4p}$. Since $n = 4p + k$ where $1 \le k \le 2p$, the graph has k vertices remaining and the vertex u_{n-p+1} has degree $2p - 1$.

$\therefore u_{n-p+1}$ cover all the remaining k vertices.

Hence, $\{u_{p-1}, u_{3p}, u_{n-p+1}\}$ is a dominating set.

Since $\left(d(u_{p-1}), d(u_{n-p+1})\right) = ((2p+1),(2p-1)) = 1$,

$\left(d(u_{p-1}), d(u_{3p})\right) = ((2p+1),(2p)) = 1$, $\left(d(u_{3p}), d(u_{n-p+1})\right) = ((2p),(2p-1)) = 1$

$\left(d(u_{p+1}), d(u_{3p}), d(u_{n-p+1})\right) = 1$, the relatively prime dominating set is $\{u_{p-1}, u_{3p}, u_{n-p+1}\}$.

$\therefore \gamma_{rpd}\left((CT(3,n))^p\right) = 3$ if $4p + 1 \le n \le 6p$ where $p \ge 2$.

Theorem 4.3. For any $CT(3,n)$ graph, $\gamma_{rpd}\left((CT(3,n))^p\right) = 0$ if $n \ge 6p + 1$ where $p \ge 2$.

Proof: Let G be a $(CT(3,n))^p$ graph. By theorem 4.2, $6p$ vertices are covered using u_p, u_{3p+1}, u_{n-p+1} vertices. But to cover the remaining vertices, we have to choose a vertex u_i of degree $2p$, where $3p + 2 \le i \le n - p$.

Then, $\left(d(u_{3p+1}), d(u_i)\right) = ((2p),(2p))) \ne 1$. Hence relatively prime dominating set does not exist. $\therefore \gamma_{rpd}\left((CT(3,n))^p\right) = 0$ if $n \ge 6p + 1$.

5 Python Program to Determine the Relatively Prime Domination Number on Power of Coconut Tree Graph

Based on the theorems in the previous sections, here is the python program to find the relatively prime domination number in power of coconut tree graph and to draw the graph.

The function CT(m,n,p) takes three value m, n, p which denotes number of pendent edges, number of vertices of the path and power of the graph(like 2,….).

Function Input

```
import math
import networkx as nx
import matplotlib.pyplot as plt
def CT(m,n,p):
    #to draw the graph
    G = nx.star_graph(m)
    P = nx.path_graph(n-1)
    CT=nx.disjoint_union(G,P)
    CT.add_edge(0,m+1)
    #get the power of CT
    power_CT=nx.power(CT,p)
    F=power_CT
    nx.draw_spring(power_CT,with_labels=True)
    plt.show()
   #to find the relatively prime domination number from the theorems
    if p>1:
        if m==2:
            if ((p+1)<=n<=((4*p)+1)):
                print("Relatively prime domination number of the resultant graph is 2")
            elif (((4*p)+2)<=n<=((6*p)+1)):
                print("Relatively prime domination number of the resultant graph is 3")
            else:
                print("Relatively prime domination number of the resultant graph is 0")
        elif m==3:
            if ((p+1)<=n<=4*p):
                print("Relatively prime domination number of the resultant graph is 2")
            elif (((4*p)+1)<=n<=6*p):
                print("Relatively prime domination number of the resultant graph is 3")
            else:
                print("Relatively prime domination number of the resultant graph is 0")
        else:
            print("Give another graph with m = 2 or 3")
```

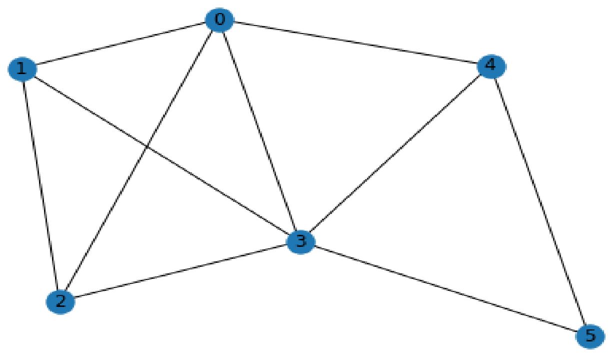
```
else:
    print("Give another value for p (p>1) to find the relatively prime domination number")
```

Output of few graphs

1. For the graph $(CT(2,4))^{2}$

Input: CT(2,4,2)

Output:

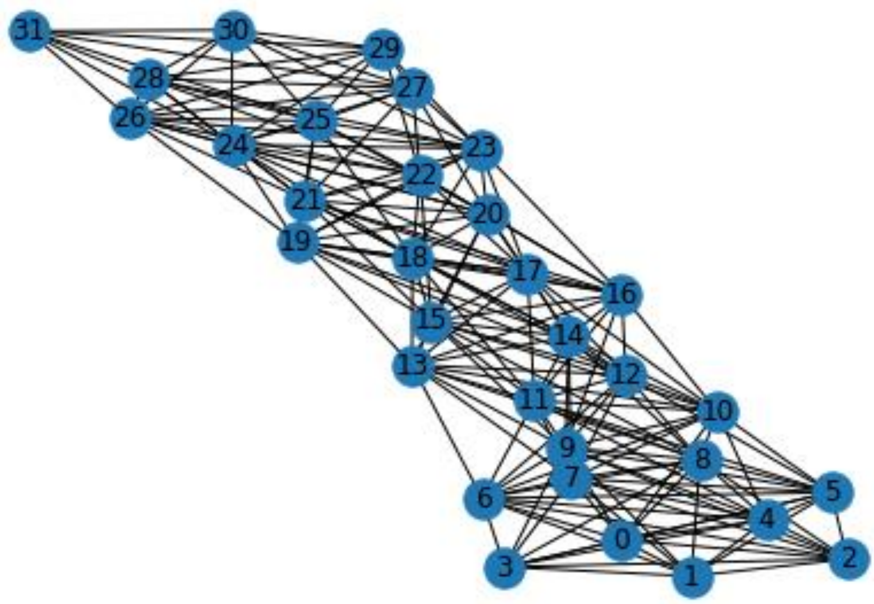


Relatively prime domination number of the resultant graph is 2

2. For the graph $(CT(3,29))^{7}$

Input: CT(3,29,7)

Output:



Relatively prime domination number of the resultant graph is 3

References

1. Harary F. Graph Theory. California: Addison-Wesley Publishing company Reading, Massachusetts. 1972.
2. Teresa W. Haynes., Stephen Hedetniemi., and Peter Slater. Fundamentals of domination in graphs. New York,.1998.
3. Berge C. Theory of Graphs and its Applications. London: Methuen. 1962.
4. Ore O. Theory of Graphs. American Mathematical Society Colloquium Publications. 1962; 38.
5. Jayasekaran C. and Jancy Vini A. 2017. Relatively Prime Dominating sets in Graphs. Annals of Pure and Applied Mathematics. 2017; 14(3): 359 - 369.